CALCULATION OF THE ACOUSTIC ADMITTANCE OF A BURNING SURFACE OF A CONDENSED SYSTEM, TAKING INTO ACCOUNT THE INSTABILITY OF PROCESSES IN THE GASEOUS PHASE

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A quasi-steady-state description of processes in the gaseous phase is used in most theoretical studies on determining the acoustic admittance of the burning surface of a condensed system (e.g. [1-6]), so that the results obtained can be used for interpreting experimental results on acoustic combustion instability at frequencies less than 10^4 Hz. An examination of the interaction of weak harmonic compression waves possessing a combustion zone, taking into account gaseous-phase time delay, valid up to frequencies of 10^4–10^5 Hz, was conducted using different models of the combustion zone in [7-9]. In this work a calculation of the acoustic admittance of the burning surface of a condensed system, taking into account gaseous-phase time delay, is performed within the framework of a two-zone model of combustion similar to the model of [7]. The variability of the combustion temperature under non-steady-state conditions, the dependence of the completeness of combustion on pressure, and the formation of entropy waves when the compression waves interact with the combustion zone is borne in mind in formulating the fundamental equations.

1. Statement of the Problem. Model of the Combustion Zone

The extent of the combustion zone even at gas-vibration frequencies reaching 10^5 Hz amounts to a small fraction of the length of an acoustic wave in gaseous combustion products. This zone may therefore be considered to be infinitely thin, coinciding with the surface of the condensed system, in analyzing the acoustic properties of a burning surface, and its acoustic properties can be described by the magnitude of the acoustic admittance

$$\zeta = -\rho_* c_* \delta u_* / \delta p \tag{1.1}$$

where δu_* is the range of variation of the discharge velocity of the gaseous combustion products from the combustion zone under the effect of harmonic compressive disturbances with amplitude δp, ρ_* is the density of the combustion products, and c_* is the sonic speed in the combustion products.

The calculation of the acoustic admittance of a burning surface reduces to determining ζ from the linearized equations that describe the reconstruction of processes in the combustion zone as the pressure varies. In solving this problem it is necessary to consider the combustion zone as extended and to make concrete assumptions regarding its structure.

A one-dimensional model of the combustion of a homogeneous condensed system is examined. It is assumed that the combustion zone has the structure schematically depicted in Fig. 1. A coordinate system bound to the condensed-phase surface, which passes through the point x = 0, is selected for writing the equations. The k phase (regions 1 and 2) is homogeneous and is characterized by a constant density ρ_1, heat capacity c_1, and coefficient of thermal conductivity λ_1. A chemical reaction, which leads to gasification of

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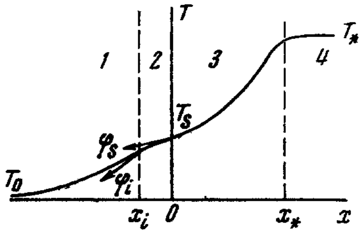

Fig. 1

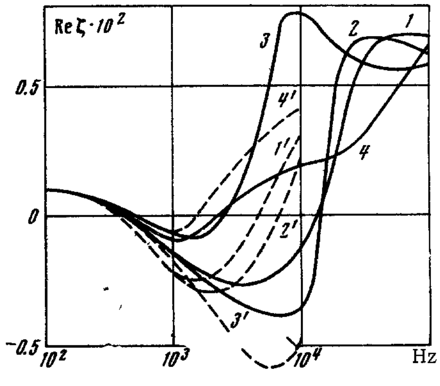

Fig. 2

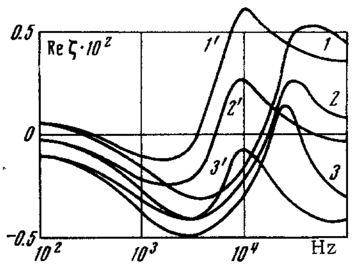

Fig. 3

the k phase, occurs in the shallow surface layer $x_i < x < 0$ (region 2). It is assumed that heat release in the k phase, under both steady and nonsteady conditions, maintains a constant value equal to Q_1. Chemical reactions do not occur in the preheating zone of the k phase, $-\infty < x < x_i$. The initial temperature of the k phase is T_0, and the surface temperature is T_s. The temperature gradients on the boundaries of the region 2 in the k phase are, respectively,

$$(dT / dx)_{x_i} \equiv \varphi_i \text{ and } (dT / dx)_{x=0} \equiv \varphi_s$$

The region $0 < x < x_*$ (region 3) is the preheating zone of the gasification products. It is assumed that the extent of the reaction zone in the gas is small in comparison to the dimensions of the preheating zone, since exothermic chemical reactions in the gas proceed in an infinitely thin region near the plane x_*, which is the flame front in the gas.

The magnitude of heat release in the gaseous-reaction zone Q_2 is assumed to depend on pressure. The region $x > x_*$ (region 4) is filled with gaseous combustion products heated to the combustion temperature T_*.

The instability of processes in each of these regions of the combustion zone can be characterized by transient periods which, for the regions 1-4, are, respectively, $\tau_1 \approx 0.3 \cdot 10^{-3}$ sec, $\tau_2 \approx 7 \cdot 10^{-6}$ sec, $\tau_3 \approx 2 \cdot 10^{-5}$ sec, and $\tau_4 \approx 10^{-6}$ sec, using approximate estimates [10]. It is evident that the instability of processes in the preheating zone in the gas must be taken into account for acoustic oscillations with frequencies up to 10^4-10^5 Hz.

2. Equations in the Preheating Gas Zone

The equations of mass, momentum, and energy conservation have the form

$$\partial \rho_2' / \partial t + \partial m' / \partial x = 0 \tag{2.1}$$

$$\partial u_2' / \partial t + u_2' \partial u_2' / \partial x = -(1 / \rho_2')(\partial p' / \partial x) \tag{2.2}$$

$$\rho_2' T_2' \left(\frac{\partial s'}{\partial t} + u_2' \frac{\partial s'}{\partial x} \right) - \frac{\partial}{\partial x} \left(\lambda_2 \frac{\partial T_2'}{\partial x} \right) = 0 \tag{2.3}$$

in the preheating region of the gaseous decomposition products of the k phase from T_s' to T_*' between the surface of the k phase ($x = 0$) and the flame front ($x = x_*$).

Values that are time dependent are denoted by primes here; ρ_2', T_2', u_2', and s' are the density, temperature, velocity, and entropy of the gas; p' is pressure ; and $m' = \rho_2' u_2'$ is the mass velocity. The heat capacities c_p and c_v and the coefficient of thermal conductivity λ_2 of the gasification products are assumed to be constant. The thermodynamic characteristics of the gasification products satisfy the equation of state of an ideal gas:

$$p' = (R\mu^{-1}) \rho_2' T_2' \tag{2.4}$$

where R is the universal gas constant, and μ is the molecular weight.

Under steady conditions it follows from (2.1)-(2.3) that

$$dm / dx = 0, \quad m = \rho_2 u_2 = \text{const} \tag{2.5}$$

$$u_2 \frac{du_2}{dx} + \frac{1}{\rho_2} \frac{dp}{dx} = 0 \tag{2.6}$$

$$\rho_2 u_2 T_2 \frac{ds}{dx} - \frac{d}{dx} \left(\lambda_2 \frac{dT_2}{dx} \right) = 0 \tag{2.7}$$

Eq. (2.6) is transformed into the form

$$\frac{d\ln p}{dx}\left[\frac{d\ln u_2}{dx}\right]^{-1} = -\gamma\frac{u_2^2}{c_2^2}, \quad \gamma = \frac{c_p}{c_v}, \quad c_2^2 = \gamma\frac{p}{\rho_2} \tag{2.8}$$

It is always true that $u_2^2/c_2^2 \ll 1$ (usually $u_2/c_2 \sim 10^{-3}$) for the combustion of condensed systems, so that we have from (2.8)

$$d\ln p\ /\ dx \ll d\ln u_2\ /\ dx \tag{2.9}$$

Using the relationship

$$\frac{ds}{dx} = \left(\frac{\partial s}{\partial T_2}\right)_p\frac{dT_2}{dx} + \left(\frac{\partial s}{\partial p}\right)_{T_2}\frac{dp}{dx} = \frac{c_p}{T_2}\frac{dT_2}{dx} - \frac{1}{\rho_2 T_2}\frac{dp}{dx}$$

we may write, by taking (2.9) into account,

$$\frac{d}{dx}\left(\lambda_2\frac{dT_2}{dx}\right) - mc_p\frac{dT_2}{dx} = 0 \tag{2.10}$$

instead of (2.7).

The boundary conditions for Eq. (2.10) on the k-phase surface ($x = 0$) have the form

$$T_2(0) = T_s, \quad \lambda_2\frac{dT_2}{dx}\bigg|_{x=0} = mc_pT_s - mc_1T_0 - mQ_1 \tag{2.11}$$

We obtain, from (2.10) and (2.11) for the temperature distribution in the heated gas layers,

$$T_2(x) = \frac{c_1T_0 + Q_1}{c_p} + \left(T_s - \frac{c_1T_0 + Q_1}{c_p}\right)\exp\left(\frac{mc_px}{\lambda_2}\right) \tag{2.12}$$

The coordinate of the flame front, where the combustion temperature T_* reaches $(c_1T_0 + Q_1 + Q_2(p))/c_p$ is given by

$$x_*(T_*) = \frac{\lambda_2}{mc_p}\ln\frac{c_pT_* - c_1T_0 - Q_1}{c_pT_s - c_1T_0 - Q_1} \tag{2.13}$$

Formulas (2.5), (2.12), and (2.13) describe the stationary distributions of a mass and temperature flow in a thermal layer in a gas occupying the region between the surface of the condensed phase, $x = 0$ (the cold boundary of the thermal layer), and the flame front in the gas, $x = x_*$ (the hot boundary of the thermal layer).

The behavior of the thermal layer in the gas in nonsteady combustion is determined by Eqs. (2.1)-(2.3). We will linearize Eqs. (2.1)-(2.3), assuming that the weak harmonic compressive disturbances $p' = p + \delta p e^{i\omega t}$, $\delta p/p \ll 1$ induce weak variations in all the values in the combustion zone, such that

$$f'(x, t) = f(x) + \delta f(x)e^{i\omega t}$$

where δf is the amplitude of the disturbance of an arbitrary characteristic variable f' of the combustion zone, and $\delta f/f \ll 1$.

Following linearization of Eq. (2.1) we obtain, by taking (2.4) into account,

$$\frac{d\delta m}{dx} = \frac{i\omega\mu p}{RT_2^2(x)}\delta T_2 - \frac{i\omega\mu}{RT_2(x)}\delta p \tag{2.14}$$

Here $T_2(x)$ is the stationary temperature distribution (2.12).

The linaerized motion equation (2.2) can be represented in the form

$$i\omega\frac{\lambda_2}{mc_pu_{20}}\frac{\delta u_2}{u_{20}} + \frac{u_2}{u_{20}}\frac{d(\delta u_2/u_{20})}{d\eta} + \frac{\delta u_2}{u_{20}}\frac{d(u_2/u_{20})}{d\eta} = -\frac{1}{\gamma M_0^2}\frac{\rho_{20}}{\rho_2}\frac{d(\delta p/p_0)}{d\eta} + \frac{1}{\gamma M_0^2}\frac{\rho_{20}^2}{\rho_2^2}\frac{\delta\rho_2}{\rho_{20}}\frac{d(p/p_0)}{d\eta} \tag{2.15}$$

Here u_{20}, p_0, and ρ_{20} are the stationary values of the gas velocity, pressure, and density at some point x_0 of the preheating zone in the gas, for example, at the surface $x = 0$ of the k phase, $\eta = mc_px/\lambda_2$, and $M_0^2 = u_{20}^2/(\gamma p_0/\rho_{20})$.

We find in this case, by estimating the order of the terms in (2.15), taking into account (2.9), the compressive disturbance in the thermal gas layer:

$$\delta p = \text{const} \tag{2.16}$$

to within terms on the order of M_0^2.

By expressing the entropy derivatives in (2.3) in terms of the temperature and pressure derivatives and linearizing the equation obtained, taking into account (2.1), (2.4), (2.9), and (2.16), we may obtain

$$\frac{d}{dx}\left[\lambda_2 \frac{d\delta T_2}{dx} - c_p T_2(x)\,\delta m - c_p m \delta T_2\right] = \frac{i\omega\mu c_v}{R}\delta p \tag{2.17}$$

Equations (2.14) and (2.17) for δm and δT_2 describe to a linear approximation thermal mass exchange in the heated layer of the gaseous combustion zone for small deviations from the steady state induced by harmonic pressure oscillations with amplitude δp, and are the fundamental equations for calculating acoustic admittance within the framework of the model assumed. It is necessary to formulate boundary conditions at the cold and hot boundaries of the thermal layer in order to solve these equations.

3. Boundary Conditions

The following conditions result if we require that the mass, thermal, and energy fluxes be continuous on the k-phase gasification surface:

$$x = 0:\ \delta m = \delta m_1,\ \delta T_2 = \delta T_s$$
$$\lambda_2(d\delta T_2 / dx) = \lambda_1 \delta\varphi_s + (c_p - c_1) m_1 \delta T_s + (c_p - c_1) T_s \delta m_1 \tag{3.1}$$

Here the values δT_s, δm_1, and $\delta\varphi_s$ are related by additional relationships which must be determined from the solution of the nonstationary-temperature-distribution problem in the k phase.

The variation in the temperature in the preheated k-phase layer is described by the equation

$$\rho_1 c_1(\partial T_1' / \partial t) + m_1' c_1(\partial T_1' / \partial x) = \lambda_1(\partial^2 T_1' / \partial x^2) \tag{3.2}$$

with boundary conditions

$$T_1'(x = -\infty) = T_0,\ T_1'(0) = T_s' \tag{3.3}$$

where T_1' is the temperature of the k phase, and m_1' is the mass-burning rate. In writing down (3.3) we have assumed that the width of the reaction zone of the k phase is small ($x_i \approx 0$), since the hot boundary of the preheating zone of the k phase (region 1) is found at the point $x = 0$ and $T_1'(x_i) = T_s'$. At the same time, when reactions are present in the k phase, we cannot ignore the difference in the temperature gradients on the inner and outer boundaries of the reaction zone of the k phase (φ_i and φ_s'). The gradients φ_i' and φ_s' are related to the energy conservation law in the region 2:

$$\lambda_1 \varphi_i' - \lambda_1 \varphi_s' = m_1' Q_1 \tag{3.4}$$

The stationary temperature distribution in the preheated layer of the k phase has the form

$$T_1(x) = T_0 + (T_s - T_0)\exp(m_1 c_1 x / \lambda_1) \tag{3.5}$$

The temperature gradient on the hot boundary of the thermal layer of the k phase is given by

$$\varphi_i = \left.\frac{dT_1(x)}{dx}\right|_{x=x_i} = \frac{m_1 c_1}{\lambda_1}(T_s - T_0)$$

The reconstruction of the temperature profile in the thermal layer of the k phase under the effect of weak compressive disturbances is described by the linearized equations (3.2) and (3.3):

$$\lambda_1 \frac{d^2\delta T_1}{dx^2} - m_1 c_1 \frac{d\delta T_1}{dx} - i\omega\rho_1 c_1 \delta T_1 = \delta m_1 \frac{c_1^2 m_1}{\lambda_1}(T_s - T_0)\exp\left(\frac{m_1 c_1}{\lambda_1} x\right) \tag{3.6}$$

$$\delta T_1(x = -\infty) = 0, \qquad \delta T_1(x = 0) = \delta T_s \tag{3.7}$$

The solution of the problem (3.6), (3.7) has the form

$$\delta T_1(x) = \left[\delta T_s - i\frac{m_1 c_1 \delta m_1}{\omega\lambda_1\rho_1}(T_s - T_0)\right]\exp\left(\frac{m_1 c_1}{2\lambda_1}\beta_1 x\right) + i\frac{m_1 c_1 \delta m_1}{\omega\lambda_1\rho_1}(T_s - T_0)\exp\left(\frac{m_1 c_1}{\lambda_1} x\right)$$
$$\beta_1 = 1 + \sqrt{1 + 4i\Omega_1}, \quad \Omega_1 = (\lambda_1\rho_1\omega)/(c_1 m_1^2) \tag{3.8}$$

By differentiating (3.8) and setting $x = 0$ we obtain a relationship between the disturbances of the temperature δT_s, mass flow δm_1, and temperature gradient $\delta\varphi_i$ at the preheated-layer–k-phase reaction zone boundary:

$$\frac{i(2-\beta_1)}{2\Omega_1}\frac{\delta m_1}{m_1}+\frac{\beta_1}{2}\frac{\delta T_s}{T_s-T_0}-\frac{\delta\varphi_i}{\varphi_i}=0 \tag{3.9}$$

We obtain after linearizing (3.4),

$$\lambda_1\delta\varphi_i-\lambda_1\delta\varphi_s=Q_1\delta m_1 \tag{3.10}$$

One more relationship between δT_s, δm_1, and $\delta\varphi_i$ is determined from an approximation analysis of the processes in a thin zone of the k phase. It is assumed that a zeroth-order chemical reaction proceeds in the reaction layer of the k phase. We may write, for the temperature in this zone,

$$\lambda_1(d^2T_1/dx^2)-m_1c_1(dT_1/dx)+\rho_1Q_1\Phi_1(T_1)=0 \tag{3.11}$$

$$x=x_i,\ T_1=T_i,\ \varphi=\varphi_i \qquad x=0,\ T_1=T_s,\ \varphi=\varphi_s \tag{3.12}$$

Here $\Phi_1(T_1)=B_1\exp(-E_1/RT_2)$ is the dependence of the chemical reaction rate on temperature, and Q_1 is the heat release, the dependence of all the values in (3.11) on time being determined in this case by the time dependence of the boundary conditions.

By approximately integrating (3.11) and (3.12) we may obtain (e.g., [11])

$$\lambda_1\varphi_i^2-\lambda_1\varphi_s^2=\frac{2\rho_1Q_1B_1\lambda_1RT_s^2}{E_1}\exp\left(-\frac{E_1}{RT_s}\right) \tag{3.13}$$

We determine, by linearizing (3.13) and using (3.10), a relationship between the amplitudes of the disturbances δm_1 of the mass-gasification rate, of the surface temperature δT_s, and of the temperature gradient at the reaction-zone–k-phase-thermal-layer-$\delta\varphi_i$ boundary in the form

$$\frac{\delta\varphi_i}{\varphi_i}+(1-q)\frac{\delta m_1}{m_1}-z_1\frac{\delta T_s}{T_s-T_0}=0 \tag{3.14}$$

$$q=\frac{Q_1}{c_1(T_s-T_0)},\quad z_1=\frac{\lambda_1\rho_1B_1}{m_1^2c_1}\exp\left(-\frac{E_1}{RT_s}\right)$$

We may represent the boundary conditions (3.1) for the functions $\delta m(x)$ and $\delta T_2(x)$, by using formulas (3.9), (3.10), and (3.14), in the form

$$x=0:\ \delta m=\delta m_1,\quad \delta T_2=G_1\delta m_1,\quad d\delta T_2/dx=G_3\delta m_1 \tag{3.15}$$

$$G_1=\frac{T_s-T_0}{m_1}\left[\frac{i(2-\beta_1)}{2\Omega_1}+1-q\right]\left[z_1-\frac{\beta_1}{2}\right]^{-1}$$

$$G_3=\frac{m_1c_1}{\lambda_2}\left(z_1-1+\frac{c_p}{c_1}\right)G_1-\frac{c_1(T_s-T_0)}{\lambda_2}\left[1+\frac{(c_1-c_p)T_s}{c_1(T_s-T_0)}\right]$$

The boundary conditions on the cold boundary of the gaseous zone (3.15) contain the mass-gasification rate disturbance δm_1, so that the solutions of the ordinary differential equations (2.14) and (2.17) under the conditions (3.15) will also contain the value δm_1, and to determine the latter it is necessary to use conditions on the hot boundary of the gaseous preheated zone.

We return to deriving the boundary conditions on the flame front in the gas, which is the hot boundary of the preheated zone and which separates the combustion products from the thermal gas layer. Under steady conditions the coordinate x_* of the flame front is determined by formula (2.13), while under nonsteady conditions the position of the flame front is time-dependent $[x'=x'(t)]$.

By considering that a stationary dependence of the mass-combustion rate m_* on the thermodynamic conditions at the flame front in the gas remains valid for small deviations from steady combustion conditions, we may write

$$\frac{\delta m_*}{m_*}=n\frac{\delta p}{p}+\frac{\varepsilon}{\tau}\frac{\delta T_*}{T_*} \tag{3.16}$$

$$n=\left(\frac{\partial\ln m_*}{\partial\ln p}\right)_{T_*},\quad \varepsilon=\left(\frac{\partial\ln m_*}{\partial T_*}\right)_p(T_s-T_0),\quad \tau=\frac{T_s-T_0}{T_*}$$

The parameters n and ε can be determined from the theoretical or experimental stationary dependence $m_*=m_*(p,T_*)$. Under steady combustion conditions, $m_1=m=m_*=\text{const}$. The mass flow m' through the flame front differs under nonsteady conditions from the mass-combustion rate m_*' in the gas. At the flame front $x_*'(t)$ we have the relationship

$$m_*'=m'(x_*')-\rho_*'dx_*'/dt \tag{3.17}$$

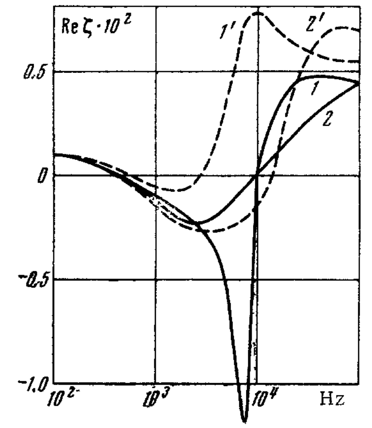

Fig. 4

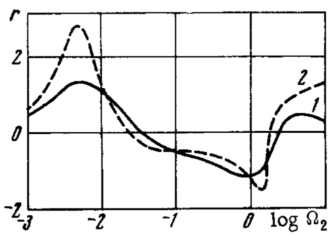

Fig. 5

We obtain after linearizing (3.17), a formula that relates the disturbance δm_* of the mass-combustion rate in the gas to the disturbance of the mass gas velocity $\delta m(x_*)$ at the flame front and the disturbance of the position of the flame front:

$$\delta m_* = \delta m\,(x_*) - i\omega\rho_*\delta x_* \tag{3.18}$$

We write the energy conservation law at the flame front under nonsteady conditions, using the definition of a flame front in a gas as a surface at which heat Q_2 is released, in the form

$$-m_*'Q_2(p') + \lambda_2 \frac{dT_2'}{dx}\Big|_{x_*'} = \lambda_2 \frac{dT_*'}{dx}\Big|_{x_*'} \tag{3.19}$$

Combustion products under steady combustion conditions have a constant temperature T_* and the right side of Eq. (3.19) vanishes. Nonuniformities in the temperature and in other values, related to the propagation of harmonic, acoustic, and entropy waves of the form $\exp(i\omega t - ikx)$, appear in the combustion products under these nonsteady conditions. The wave numbers of these waves are as follows (e.g., [12]:

$$k^- = i\frac{\omega}{c_*}(1 + M_*), \quad k^+ = -i\frac{\omega}{c_*}(1 - M_*) \quad \left(M_* = \frac{u_*}{c_*}\right)$$
$$k^{(0)} = \frac{\omega}{u_*}\frac{1 - \sqrt{1 + 4i\Omega_2}}{2\Omega_2}, \quad \Omega_2 = \frac{\lambda_2\rho_*}{c_p m_*^2}\,\omega \tag{3.20}$$

Values related to acoustic waves propagating in the positive and negative directions of the x axis are denoted by the subscripts + or –, respectively, while values related to an entropy wave are denoted by the subscript 0. Relationships between the disturbances in acoustic and entropy waves, described to within terms on the order of M_*^2, have the form

$$\delta u_*^{\pm} = \pm\frac{c_*}{\gamma}\frac{\delta p^{\pm}}{p}, \quad \frac{\delta\rho_*^{\pm}}{\rho_*} = \frac{1}{\gamma}\frac{\delta p^{\pm}}{p}, \quad \frac{\delta T_*^{\pm}}{T_*} = \frac{\gamma - 1}{\gamma}\frac{\delta p^{\pm}}{p}$$
$$\delta u_*^{(0)} = 0, \quad \delta p^{(0)} = 0, \quad \delta T_*^{(0)}/T_* = -\delta\rho_*^{(0)}/\rho_* \tag{3.21}$$

The distrubances of all the thermodynamic parameters and the gas velocity in the combustion products are represented as the sum of the disturbances transmitted by each of the waves, in particular

$$\delta p = \delta p^+ + \delta p^-, \quad \delta m\,(x_*) = \rho_*(\delta u_*^+ + \delta u_*^-) + u_*(\delta\rho_*^+ + \delta\rho_*^- + \delta\rho_*^{(0)}), \quad \delta T_* = \delta T_*^+ + \delta T_*^- + \delta T_*^{(0)} \tag{3.22}$$

We obtain from (3.22), by taking into account that the temperature disturbances are of a wave character, that

$$d\delta T_*/dx = k^+\delta T_*^+ + k^-\delta T_*^- + k^{(0)}\delta T_*^{(0)} \tag{3.23}$$

We find by linearizing (3.19) and taking into account (2.12) and (3.20)-(3.23), that

$$\delta m_* Q_2(p) + m\frac{c_pT_*}{p}\alpha\delta p - \lambda_2\frac{d\delta T_2}{dx}\Big|_{x_*} - \frac{m^2c_p}{\lambda_2}Q_2(p)\,\delta x_* = -\lambda_2 k^{(0)}\left(\delta T_* - \frac{\gamma - 1}{\gamma}\frac{T_*}{p}\delta p\right) \tag{3.24}$$

The quantity

$$\alpha = (dQ_2/dp)\,(p/c_p\,T_*)$$

characterizes the variation in the completeness of combustion as the pressure is varied.

We obtain by using the given combustion temperature, $T_*' = T_2'(x_*')$ and formula (2.12), that

$$\delta T_* = \delta T_2(x_*) + (mQ_2(p)\lambda_2^{-1})\delta x_* \tag{3.25}$$

We obtain the boundary condition for the functions $\delta m(x)$ and $\delta T_2(x)$ on the surface $x = x_*$ by eliminating δm_*, δT_*, and δx_* from (3.16), (3.18), (3.24), and (3.25):

$$\left(Q_2(p) + \frac{A}{m}\right)\delta m\,(x_*) + \left(\lambda_2 k^{(0)} - \frac{\varepsilon}{\tau}\frac{A}{T_*}\right)\delta T_2(x_*) - \lambda_2\frac{d\delta T_2}{dx}\Big|_{x_*} + \left(\frac{mc_pT_*}{p}\alpha - \lambda_2\frac{\gamma - 1}{\gamma}\frac{T_*}{p}k^{(0)} - n\frac{A}{p}\right)\delta p = 0 \tag{3.26}$$
$$A = (mk^{(0)} - i\omega\rho_* - m^2c_p/\lambda_2)\left(\frac{i\omega\rho_*}{m} + \frac{\varepsilon}{\tau}\frac{mQ_2}{\lambda_2 T_*}\right)^{-1} Q_2$$

Condition (3.26) allows us to determine the value δm_1 of the mass-velocity disturbance on the surface of the k phase, which is the eigenvalue of the problem, as well as to obtain the complete solution of the nonsteady reconstruction problem of the thermal gas layer under the effect of a compressive disturbance δ_p.

4. Results of the Calculations and Discussion

We obtain from (1.1) using the relationships (3.21) and (3.22) the following for the acoustic admittance of a hot surface of a condensed system:

$$\zeta = -\gamma \frac{u_*}{c_*}\left[\frac{\delta m(x_*)}{m} \Big/ \frac{\delta p}{p} + \frac{\delta T_*}{T_*} \Big/ \frac{\delta p}{p} - 1\right] \tag{4.1}$$

Further, expressing δT_* in terms of $\delta m(x_*)$ and $\delta T_2(x_*)$ we obtain from (4.1) using Eqs. (3.16), (3.18), and (3.25),

$$\zeta = -\gamma \frac{u_*}{c_*}\left[\left(1+\frac{1}{\Lambda}\right)\frac{\delta m(x_*)}{m} \Big/ \frac{\delta p}{p} + \left(1-\frac{\varepsilon}{\tau\Lambda}\right)\frac{\delta T_2(x_*)}{T_*} \Big/ \frac{\delta p}{p} - \frac{n}{\Lambda} - 1\right]$$
$$\Lambda = \frac{i\Omega_2 c_p T_*}{Q_2(p)} + \frac{\varepsilon}{\tau} \tag{4.2}$$

In (4.2) the values

$$\frac{\delta m(x_*)}{m} \Big/ \frac{\delta p}{p} \text{ and } \frac{\delta T_2(x_*)}{T_*} \Big/ \frac{\delta p}{p}$$

are determined by solving the problem (2.14), (2.17), (3.15), (3.26). An acoustic wave is amplified when reflected from the combustion surface of a condensed system of the real part $\mathrm{Re}\,\zeta$ of the acoustic admittance is negative.

The calculation of the real part $\mathrm{Re}\,\zeta$ of the acoustic admittance reduces to the numerical integration of a system of four first-order ordinary differential equations in real variables on the segment $[0, x_*]$ with boundary conditions (3.15), (3.26) at the endpoints. Integration conducted by the Runge–Kutta method was begun from the point $x = 0$ with the conditions (3.15), and the condition (3.26) was satisfied at the right end point $x = x_*$.

If condition (3.26) is not satisfied, the value of δm_1 at $x = 0$ is changed and integration is repeated for a new value of δm_1. Results are presented below of a calculation of the dependence of $\mathrm{Re}\,\zeta$ on frequency according to Eq. (4.2) for the following values of the parameters: $\lambda_1 = \lambda_2 = 5 \cdot 10^{-4}$ cal/sec·cm·°K, $c_1 = c_p = 0.33$ cal/g·°K, $\rho_1 = 1.6$ g/cm^3, $m = 2$g/sec·cm^2, $p = 50$ atm, $\mu = 29$, $T_0 = 300°$ K, $T_s = 600°$ K, $\varepsilon = 1$, $\gamma = 1.25$, $z_1 = 10$, $Q_1 + Q_2 = 800$ cal/g. In pace of the variable n we used the parameter $\nu = (\partial \ln m_* / \partial \ln p)_{T_0}$, related to n by the relation $\nu = n + \varepsilon\tau^{-1}\alpha$. A value of $\nu = 0.67$ was assumed in the calculations.

The dependence of $\mathrm{Re}\,\zeta$ on frequency for different values of heat release Q_1 in the k phase ($\alpha = 0$) is depicted in Fig. 2 by solid curves. Results of a calculation of the dependence of $\mathrm{Re}\,\zeta$ on frequency for the same values of the parameters but without taking into account gaseous-phase time delay [2] are depicted by dotted curves for comparison. Values for Q_1 of 20 cal/g, 40 cal/g, and 80 cal/g correspond to curves 1, 1'; 2, 2'; and 3, 3', respectively. Curves 4, 4' were obtained for the case of an endothermic reaction in the k phase with $Q_1 = -80$ cal/g. It is evident that the influence of gaseous-phase time delay is appreciable beginning with frequencies on the order of 10^3 Hz. A calculation of gaseous-phase time delay can lead to either an increase or a decrease in the tendency of the condensed system towards acoustic-combustion instability. The gaseous-phase time delay also leads to the appearance of a maximum of $\mathrm{Re}\,\zeta$ at high frequencies, which corresponds to the greatest attenuation of the acoustic waves when reflected, which shifts towards lower frequencies with increasing Q_1.

In Fig, 3 the dependence of $\mathrm{Re}\,\zeta$ on frequency for different values of α (curves 1, 1' correspond to $\alpha = 0.1$, curves 2, 2' correspond to $\alpha = 0.3$, and curves 3, 3' correspond to $\alpha = 0.5$) and of heat release Q_1 in the k phase (curves 1–3 correspond to $Q_1 = 20$ cal/g, while curves 1'–3' correspond to $Q_1 = 80$ cal/g) is shown. The tendency of the condensed system toward amplification of the acoustic compression waves increases over the entire frequency band with increasing α.

In this work a boundary condition on the surface of the condensed phase, which takes into account to a quasi-steady-state approximation the extent of the chemical reaction zone in the k phase that leads to gasification, was used for calculating the size of $\mathrm{Re}\,\zeta$. In a number of other studies (e. g., [13]) it was assumed that the passage of the k phase into a gas is a surface process whose mass rate is determined by the law

$$m_1 \sim \exp\left(-E_s / RT_s\right)$$

For this model we have, in place of the condition (3.14) on the surface of the condensed phase,

$$\delta m_1 / m_1 = (E_s / RT_s^2)\delta T_s \tag{4.3}$$

Dependences of Re ζ on frequency, calculated using conditions on the surface of the condensed phase in the form (3.14) and in the form (4.3), are presented in Fig. 4 by solid curves and dotted curves, respectively, for comparison. A value for Q_1 of 80 cal/g corresponds to the curves 1, 1', while a value for Q_1 of 20 cal/g corresponds to the curves 2, 2'.

A calculation was performed in [9] of the acoustic admittance of the hot surface of a condensed system, taking into account the spatial extent of the simple chemical reaction zone in the gas. Dependences are presented in Fig. 5 of the real part $r = -c_* \mathrm{Re}\,\zeta / \gamma u_*$ of a hot surface on log Ω_2 as calculated in [9] (dotted curve 2) and in this work (solid curve 1), given identical values of the parameters of the condensed system.

It is evident that, by taking into account the distributive nature of the chemical reaction in the gas, we arrive at more expressed extrema on the acoustic admittance–frequency curve. High qualitative and satisfactory quantitative agreement between the curves is observed.

In conclusion, we note that a simplified scheme and approximate values for the kinetic constants of the chemical reactions were used in the calculation conducted. Therefore, results of the calculation can be used only for explaining the qualitative features of the dependence of acoustic admittance on different characteristics of a combustion process (in particular, on heat release in the k phase), which corresponds to the contemporary state of experimental investigation in the field of acoustic combustion instability.

LITERATURE CITED

1. F. A. Williams, "Response of a burning solid to small-amplitude pressure oscillations," J. Appl. Phys., 33, No. 11 (1962).
2. S. S. Novikov and Yu. S. Ryazantsev, "Interaction of acoustic waves with the hot surface of condensed systems," Zh. Prikl. Mekhan. Tekh. Fiz., No. 2 (1966).
3. H. Krier, J. S. T'ien, W. A. Sirignano, and M. Summerfield, "Nonsteady burning phenomena of solid propellants: Theory and experiments," AIAA J., 6, No. 2 (1968).
4. F. E. Culick, " A review of calculations for unsteady burning of a solid propellant," AIAA J., 6, No. 12 (1968).
5. S. S. Novikov, Yu. S. Ryazantsev, and V. E. Tul'skikh, "Acoustic admittance of a hot powdery surface," Zh. Prikl. Mekhan. Tekh. Fiz., No. 5 (1969).
6. Yu. A. Gostintsev, L. A. Sukhanov, and P. F. Pokhil, "Theory of nonsteady combustion of powder. Combustion under harmonically varying pressure," Zh. Prikl. Mekhan. Tekh. Fiz., No. 5 (1971).
7. W. R. Hart and F. T. McClure, "Combustion instability: Acoustic interaction with a burning propellant surface," J. Chem. Phys., 30, No. 6 (1959).
8. V. P. Volkov and Yu. I. Medvedev, "Interaction of acoustic waves with the hot surface of solid fuels at increased frequencies," Zh. Prikl. Mekhan. Tekh. Fiz., No. 1 (1969).
9. J. S. T'ien, "Oscillatory burning of solid propellants including gas-phase time lag," Combust. Sci., Technol., 5, No. 2 (1972).
10. F. T. McClure, R. W. Hart, and J. F. Board, "Solid-fuel rocket engines as sources of acoustic oscillations," in: Investigation of Solid-Fuel Rocket Engines [in Russian], IL, Moscow (1963).
11. S. S. Novikov and Yu. S. Ryazantsev, "Theory of stationary propagation velocity of the front of an exothermic reaction in a condensed medium," Zh. Prikl. Mekhan. Tekh. Fiz., No. 3 (1965).
12. R. W. Hart and R. H. Cantrell, "Amplification and attenuation of sound by burning propellants," AIAA J., 1, No. 2 (1963).
13. M. R. Denison and E. Baum, "A simplified model of unstable burning in solid propellants," ARS J., 31, No. 8 (1961).